MAΘ Competition Team Homework Set 3

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Problem 1. Russell and Anders are playing a game where they take turns removing either 1 or 3 coins from a pile of 20. The person to take the last coin loses. If both players play optimally and Russell goes first, can he win?

Problem 2. There is a pile of 100 stones, and Alice and Bob alternate taking either 1, 3, or 4 stones from the pile at a time. A player loses if they cannot make a move. If Alice goes first, who has a winning strategy?

Problem 3. In a game, Bob goes first, and he has to say a positive integer less than or equal to 16. Then, Allison must add a positive integer less than or equal to 16 to Bob's number, at which point Bob must add a positive integer less than or equal to 16, and so on. The winner is whomever says the number 2015. What number must Bob say first to ensure that he will win the game if both players play optimally?

Problem 4. Jiwu and Yumi play the following game on a 20×20 chessboard: they take turns playing down 3×1 rectangles on the board such that the rectangles cover exactly three squares of the chessboard that were not previously covered by some move. The first player with no legal move loses. If Jiwu moves first, determine who has a winning strategy. Explain.

Problem 5. Ashley and Elizabeth play a game. There are 9 cards numbered 1 through 9 on a table, and the players alternate taking the cards, with Ashley going first. A player wins if at any point they hold three cards with sum 15; if all nine cards are taken before this occurs, the game is a tie. Does either player have a winning strategy? Explain.