MAΘ Competition Team Homework Set 5

Anders Christensen, Hannah Kim

November 7, 2025

Due date: November 13, 2025

Problem 1. State and prove the formula for infinite geometric series.

Problem 2. What is the value of $1 - \sum_{k=1}^{\infty} \frac{1}{8^k}$?

Problem 3. The sum of the first 2011 terms of a geometric sequence is 200. The sum of the first 4022 terms is 380. Find the sum of the first 6033 terms.

Problem 4. Two geometric sequences $a_1, a_2, a_3, \ldots$ and $b_1, b_2, b_3, \ldots$ have the same common ratio, with $a_1 = 27$, $b_1 = 99$, and $a_{15} = b_{11}$. Find a_9.

Problem 5. For $-1 < r < 1$, let $S(r)$ denote the sum of the geometric series $12 + 12r + 12r^2 + 12r^3 + \ldots$.Let a between -1 and 1 satisfy $S(a)S(-a) = 2016$. Find $S(a) + S(-a)$.

Problem 6. Evaluate the following sum.

$$\sum_{k=1}^{\infty} \frac{k^2}{2^k}$$

Problem 7 (Challenge). In terms of $p = \sum_{k=1}^{\infty} \frac{1}{k^2}$ and $q = \sum_{k=1}^{\infty} \frac{1}{k^3}$, express the following.

$$\sum_{j=1}^{\infty} \sum_{k=1}^{\infty} \frac{1}{(j+k)^3}$$

Problem 8 (Challenge). Find $\sum_{n=0}^{\infty} \frac{\sin(nx)}{3^n}$ if $\sin(x) = 1/3$ and $0 \leq x \leq \pi/2$.