MAΘ Competition Team HW Set 8

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Problem 1. Compute

$$\sum_{d|60} \varphi(d)$$

(note: | is the "is a divisor of" sign, meaning that d is a divisor of 60.)

Problem 2. Find the remainder when

$$3^{2026} + 5^{2026}$$

is divided by 7.

Problem 3. Let p be an odd prime. Define

$$S = \sum_{k=1}^{p-1} k^{p-1}$$

Prove that

$$S \equiv -1 \pmod{p}.$$

Problem 4. Let x be the least real number greater than 1 such that $\sin(x) = \sin(x^2)$, where the arguments are in degrees. What is x rounded up to the closest integer?

Problem 5. An integer N is selected at random in the range $1 \leq N \leq 2020$. What is the probability that the remainder when N^{16} is divided by 5 is 1?

Problem 6. Let n be the smallest positive integer such that

$$7^n \equiv 1 \pmod{100}.$$

Compute the value of

$$\varphi(n) + n,$$

where φ denotes Euler's totient function.

Problem 7 (Challenge). Find the number of rational numbers r, $0 < r < 1$, such that when r is written as a fraction in lowest terms, the numerator and the denominator have a sum of 1000.