MAΘ Competition Team Homework Set 7

Anders Christensen, Minjoo Kim

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Problem 1. Find the sum of all entries in row 12 of Pascal's Triangle.

Problem 2. Find the coefficient of x^5 in the expansion of

$$(2x-3)^8$$

Problem 3. Evalute:

$$\sum_{k=0}^{n}\sum_{j=0}^{k}\sum_{i=0}^{j}\binom{i}{r}$$

Express your answer as a single binomial coefficient.

Problem 4. Prove the Hockey Stick Identity using Pascal's Triangle (hint: Use Pascal's Identity and recursion).

Problem 5. In which row of Pascal's Triangle do three consecutive entries occur that are in the ratio $3:4:5$?

Problem 6. The polynomial $1-x+x^2-x^3+\cdots+x^{16}-x^{17}$ may be written in the form $a_0+a_1y+a_2y^2+\cdots+a_{16}y^{16}+a_{17}y^{17}$, where $y=x+1$ and the a_i's are constants. Find the value of a_2.

Problem 7. A triangular array of squares has one square in the first row, two in the second, and in general, k squares in the kth row for $1 \leq k \leq 11$. With the exception of the bottom row, each square rests on two squares in the row immediately below (illustrated in given diagram). In each square of the eleventh row, a 0 or a 1 is placed. Numbers are then placed into the other squares, with the entry for each square being the sum of the entries in the two squares below it. For how many initial distributions of 0's and 1's in the bottom row is the number in the top square a multiple of 3?