

# MAΘ Competition Team Homework Set 1

Anders Christensen, Hannah Kim

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**Problem 1.** Solve  $xy + 5x + 7y + 40 = 0$  over the integers.

**Problem 2.** Explain Simon's Favorite Factoring Trick in your own words.

**Problem 3.** An  $m \times n \times p$  rectangular box has half the volume of an  $(m+2) \times (n+2) \times (p+2)$  rectangular box, where  $m$ ,  $n$ , and  $p$  are integers, and  $m \leq n \leq p$ . What is the largest possible value of  $p$ ?

**Problem 4.** If  $kn + 54k + 2n + 108$  has a remainder of 3 when divided by 5, and  $k$  has a remainder of 1 when divided by 5, find the value of the remainder when  $n$  is divided by 5.

**Problem 5.** Factor  $x^4 + 1$  into two polynomials with real coefficients (Hint:  $x^2$ )

**Problem 6.** If  $a + b + c = 0$ , prove that  $a^3 + b^3 + c^3 = 3abc$

**Problem 7. (Challenge)** The integer  $N$  is positive. There are exactly 2005 ordered pairs  $(x, y)$  of positive integers satisfying

$$\frac{1}{x} + \frac{1}{y} = \frac{1}{N}$$

Prove that  $N$  is a perfect square.