MAΘ Competition Team HW Set 12

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Problem 1. How many solutions does the equation $v + w + x + y + z = 21$ have, where v, w, x, y, z are all nonnegative integers?

Problem 2. How many cubic polynomials $f(x)$ with positive integer coefficients are there such that $f(1) = 9$?

Problem 3. How many ways can I distribute 20 pieces of candy to 7 children if each child must receive at least one piece, and two of the children are twins that insist on receiving the same amount of candy?

Problem 4. Mr. Sydow is giving out 15 pieces of candy to Sean, Chong, Eric, and Russell, with each getting at least one piece. However, Chong insists that he must receive more pieces than everybody else. How many ways can Mr. Sydow distribute the candy? (Hint: You can use the Hockey Stick Identity at some point...)

Problem 5. A hexomino game piece has six numbers on it from 1 to 9. Two pieces are considered different if they have different numbers; in other words, the order of the numbers on the piece doesn't matter. For example, two hexominos with numbers {1,2,3,4,5,6} and {1,3,2,4,6,5} clockwise are considered the same. A piece may have a number repeated, or have the same number in all six positions. How many distinct hexominos are there?

Problem 6. How many 15-digit base 4 numbers are there with eight 0's that appear in 3 groups? (For example, 230001330210000 is one such numbers; the 3 groups are 000, 0, and 0000).

Problem 7 (Challenge). In a certain lottery, 7 balls are drawn at random from n balls numbered 1 through n. If the probability that no pair of consecutive numbers is drawn equals the probability of drawing exactly one pair of consecutive numbers, find n.