

MA Θ Competition Team HW Set 13

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Problem 1. Find the diagonal length of an isosceles trapezoid with bases of lengths 8 and 20 and legs of length 10.

Problem 2. Quadrilateral $ABCD$ with consecutive sides of 8, 15, and 12 is inscribed in a circle with circumference 17π . Given that AC is a diameter of the circle, what is the length of the other diagonal of the quadrilateral?

Problem 3. Inscribed in a circle is a quadrilateral having sides of length 25, 39, 52, and 60 taken consecutively. What is the diameter of this circle? (Show work using Ptolemy's)

Problem 4. Prove that a trapezoid is cyclic if and only if it is isosceles.

Problem 5. In a circle, parallel chords of lengths 2, 3, and 4 determine central angles of α , β and $\alpha + \beta$ radians, respectively, where $\alpha + \beta < \pi$. If $\cos \alpha$, which is a positive rational number, is expressed as a fraction in lowest terms, what is the sum of its numerator and denominator?

Problem 6 (Challenge). In a certain circle, the chord of a d -degree arc is 22 centimeters long, and the chord of a $2d$ -degree arc is 20 centimeters longer than the chord of a $3d$ -degree arc, where $d < 120$. The length of the chord of a $3d$ -degree arc is $-m + \sqrt{n}$ centimeters, where m and n are positive integers. Find $m + n$.

Problem 7 (Challenge). A circle with center O has radius 25. Chord \overline{AB} of length 30 and chord \overline{CD} of length 14 intersect at point P . The distance between the midpoints of the two chords is 12. The quantity OP^2 can be represented as $\frac{m}{n}$, where m and n are relatively prime positive integers. Find the remainder when $m + n$ is divided by 1000.