MAΘ Competition Team Problem Set 2

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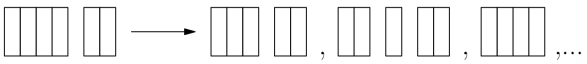
Problem 1. Hammurabi and Nebuchadnezzar are playing a game where they take turns removing either 1 or 2 coins from a pile of 10. The person to take the last coin loses. If both players play optimally and Hammurabi goes first, can he win?

Problem 2. Bela and Jenn play the following game on the closed interval $[0, n]$ of the real number line, where n is a fixed integer greater than 4. They take turns playing, with Bela going first. At his first turn, Bela chooses any real number in the interval $[0, n]$. Thereafter, the player whose turn it is chooses a real number that is more than one unit away from all numbers previously chosen by either player. A player unable to choose such a number loses. Using optimal strategy, which player will win the game?

Problem 3. Barbara and Jenna play the following game, in which they take turns. A number of coins lie on a table. When it is Barbara's turn, she must remove 2 or 4 coins, unless only one coin remains, in which case she loses her turn. When it is Jenna's turn, she must remove 1 or 3 coins. A coin flip determines who goes first. Whoever removes the last coin wins the game. Assume both players use their best strategy. Who will win when the game starts with 2013 coins and when the game starts with 2014 coins?

Problem 4. Arjun and Beth play a game in which they take turns removing one brick or two adjacent bricks from one "wall" among a set of several walls of bricks, with gaps possibly creating new walls. The walls are one brick tall. For example, a set of walls of sizes 4 and 2 can be changed into any of the following by one move: $(3, 2), (2, 1, 2), (4), (4, 1), (2, 2)$, or $(1, 1, 2)$.



Problem 5. Two players play a game on an 8×8 chessboard. A rook begins on the upper-left corner square $(1, 1)$. On each turn, a player must move the rook any positive number of squares either right or down, but not both. The rook may not move left or up, and moving zero squares is not allowed. The player who moves the rook to the bottom-right corner square $(8, 8)$ wins the game. If both players play optimally and the first player starts, who will win the game?