MAΘ Competition Team Problem Set 4

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Problem 1. Prove by induction that $n^3 + 2n$ is divisible by 3 for every $n \geq 1$.

Problem 2. Prove by induction that $1 + 2 + 3 + ... + n = \frac{(n)(n+1)}{2}$ for every positive integer n.

Problem 3. Bernoulli's inequality asserts that if $a \in \mathbb{R}$, $a > -1$ and $n \in \mathbb{N}$, $n \geq 2$, then $(1 + a)^n > 1 + an$. Prove, by induction, the validity of Bernoulli's identity.

Problem 4 (COMC). Let F be a function which maps integers to integers by the following rules: $F(n) = n - 3$ if $n \geq 1000$; $F(n) = F(F(n + 5))$ if $n < 1000$.
Show that $F(984) = F(F(F(1004)))$.

Problem 5. $f(n) = 3^{2n+4} - 2^{2n}$, $n \in \mathbb{N}$. Prove by induction that $f(n)$ is divisible by 5, for all $n \in \mathbb{N}$.

Problem 6 (Challenge). Prove by induction that $1^4+2^4+3^4+...+n^4 = \frac{(n)(n+1)(2n+1)(3n^2+3n-1)}{30}$ for every positive integer n.