MAΘ Competition Team Problem Set 6

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Problem 1. What is $(34+33i)\left(\overline{34+33i}\right)$?

Problem 2. Find $\left|\dfrac{7-24i}{4+3i}\right|$.

Problem 3. There are 24 different complex numbers z such that $z^{24}=1$. For how many of these is z^6 a real number?

Problem 4. Define a sequence of complex numbers by $z_1=0$, $z_{n+1}=z_n^2+i$ for $n\geq 1$. How far away from the origin is z_{111}?

Problem 5. Suppose that x, y, and z are complex numbers such that $xy=-80-320i$, $yz=60$, and $zx=-96+24i$, where $i=\sqrt{-1}$. Then there are real numbers a and b such that $x+y+z=a+bi$. Find a^2+b^2.

Problem 6 (Challenge). Write $\cos 5\theta$ as a function of $\cos\theta$.

Problem 7 (Challenge). Find all ordered pairs of real numbers (a,b) such that $a+\dfrac{a+8b}{a^2+b^2}=2$ and $b+\dfrac{8a-b}{a^2+b^2}=0$.