MAΘ Competition Team Problem Set 4

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Problem 1. The first three terms of a geometric series are $(k+4)$, k and $(2k–15)$ respectively, where k is a positive constant. Find the value of k and the common ratio of this series.

Problem 2. Anders, Hannah, and Jiwu are sharing a large block of cheese. They take turns cutting off half of what remains and eating it: first Anders eats half of the cheese, then Hannah eats half of the remaining half, then Jiwu eats half of what remains, then back to Anders, and so on. They stop when the cheese is too small to see. About what fraction of the original block of cheese does Anders eat in total?

Problem 3. An infinite geometric series has sum 2005. A new series, obtained by squaring each term of the original series, has 10 times the sum of the original series. Find the common ratio of the original series.

Problem 4. The sum of an infinite geometric series is a positive number S, and the second term in the series is 1. What is the smallest possible value of S? (Hint: AM-GM)

Problem 5. The increasing geometric sequence $x_0, x_1, x_2, \ldots$ consists entirely of integral powers of 3. Find $\log_3(x_{14})$ given that

$$\sum_{n=0}^{7} \log_3(x_n) = 308,$$

$$56 \leq \log_3\left(\sum_{n=0}^{7} x_n\right) \leq 57$$

Problem 6 (Challenge). Given that $\cos(\theta) = 1/5$, evaluate the following.

$$\sum_{n=0}^{\infty} \frac{\cos(n\theta)}{2^n}$$