MAΘ Competition Team Problem Set 8

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Problem 1. Find the GCD and LCM of 96 and 18.

Problem 2. For two positive integers m and n, suppose $\text{GCD}(m, n) \cdot \text{LCM}(m, n) = 2016$. What is the minimum possible value of $m + n$?

Problem 3. Prove that the fraction $\frac{21n+4}{14n+3}$ is irreducible for every positive integer n.

Problem 4. Find all pairs of positive integers (a, b) such that $\text{LCM}(a, b) - \text{GCD}(a, b) = \frac{ab}{5}$.

Problem 5. How many positive integers n are there such that n is a multiple of 5, and the LCM of 5! and n equals 5 times the GCD of 10! and n?

Problem 6 (Challenge). Let n be the least positive integer greater than 1000 for which $\text{GCD}(63, n + 120) = 21$ and $\text{GCD}(n + 63, 120) = 60$. Find n.