MAΘ Competition Team Problem Set 12

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Problem 1. Prove the divisibility rules of positive integers 1∼10.

Problem 2. Check if the number 16387 is divisible by 7 and 11.

Problem 3. How many 4-digit positive integers (that is, integers between 1000 and 9999, inclusive) having only even digits are divisible by 5?

Problem 4. Call a positive integer an uphill integer if every digit is strictly greater than the previous digit. For example, $1357, 89$, and 5 are all uphill integers, but $32, 1240$, and 466 are not. How many uphill integers are divisible by 15?

Problem 5. Let $S(n)$ equal the sum of the digits of positive integer n. For example, $S(1507) = 13$. For a particular positive integer n, $S(n) = 1274$. What is the remainder when S(n+1) is divided by 9?

Problem 6. There are 8!= 40320 eight-digit positive integers that use each of the digits 1,2,3,4,5,6,7,8 exactly once. Let N be the number of these integers that are divisible by 22. Find the difference between N and 2025.