

# MA $\Theta$ Competition Team Problem Set 15

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**Problem 1.** Determine the equation of the perpendicular bisector of the line segment passing through the points  $(-2, -5)$  and  $(2, 7)$ .

**Problem 2.** If the points  $A(1, -2)$ ,  $B(2, 3)$ ,  $C(-4, -3)$  and  $D$  form a parallelogram, find one possible coordinate of  $D$ .

**Problem 3.** Square  $ABCD$  is positioned in the first quadrant with  $A$  on the  $y$ -axis,  $B$  on the  $x$ -axis, and  $C = (13, 8)$ . Compute the area of the square.

**Problem 4.** Let points  $A = (0, 0)$ ,  $B = (1, 2)$ ,  $C = (3, 3)$ , and  $D = (4, 0)$ . Quadrilateral  $ABCD$  is cut into equal area pieces by a line passing through  $A$ . This line intersects  $\overline{CD}$  at point  $(p, q)$ . Find  $(p, q)$ .

**Problem 5.** In the right triangle  $\triangle ACE$ , we have  $AC = 12$ ,  $CE = 16$ , and  $EA = 20$ . Points  $B$ ,  $D$ , and  $F$  are located on  $AC$ ,  $CE$ , and  $EA$ , respectively, so that  $AB = 3$ ,  $CD = 4$ , and  $EF = 5$ . What is the ratio of the area of  $\triangle DBF$  to that of  $\triangle ACE$ ?

**Problem 6.** Triangle  $ABC$  lies in the cartesian plane and has an area of 70. The coordinates of  $B$  and  $C$  are  $(12, 19)$  and  $(23, 20)$ , respectively, and the coordinates of  $A$  are  $(p, q)$ . The line containing the median to side  $BC$  has slope  $-5$ . Find the largest possible value of  $p + q$ .