MAΘ Competition Team Trial Set

Russell Jin, Anders Christensen, Hannah Kim

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Problem 1. What is the value of the sum $1 + \frac{1}{3} + \frac{1}{9} + \frac{1}{27} + ...$?

Problem 2. Positive integers m and n are both greater than 50, have an LCM equal to 480, and have a GCD equal to 12. Find $m + n$.

Problem 3. The isosceles right triangle ABC has right angle at C and area 12.5. The rays trisecting $\angle ACB$ intersect AB at D and E. What is the area of $\triangle CDE$?

Problem 4. Two jokers are added to a 52 card deck and the entire stack of 54 cards is shuffled randomly. What is the expected number of cards that will be strictly between the two jokers?

Problem 5. In triangle ABC we have $AB = 7$, $AC = 8$, $BC = 9$. Point D is on the circumscribed circle of the triangle so that AD bisects angle BAC. What is the value of $\frac{AD}{CD}$?

Problem 6. Let $N = 123456789101112\ldots4344$ be the 79-digit number that is formed by writing the integers from 1 to 44 in order, one after the other. What is the remainder when N is divided by 45?

Problem 7. The equation $z^6 + z^3 + 1 = 0$ has one complex root with argument (angle) between 90° and 180° in the complex plane. Determine the degree measure of θ.

Problem 8. Let p, q, and r be solutions to $3x^3 - 4x^2 + 5x + 7 = 0$. Find $\frac{1}{p} + \frac{1}{q} + \frac{1}{r}$.

Problem 9. Solve the following system in real numbers,

$$\sqrt{xy}(3x + y)(3y + x) = 14,$$
$$(x + y)(x^2 + 14xy + y^2) = 36.$$

(Notice the symmetry between x and y.)