MAΘ Competition Team Problem Set 1

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Problem 1. Factor $xy + 3x + 4y + 12$.

Problem 2. If $kn + 54k + 2n + 108$ has a remainder of 3 when divided by 5, and k has a remainder of 1 when divided by 5, find the value of the remainder when n is divided by 5.

Problem 3. An $m \times n \times p$ rectangular box has half the volume of an $(m+2) \times (n+2) \times (p+2)$ rectangular box, where m, n, and p are integers, and $m \leq$n p.$Whatisthelargestpossiblevalueof$p?

Problem 4. If $kn + 54k + 2n + 108$ has a remainder of 3 when divided by 5, and k has a remainder of 1 when divided by 5, find the value of the remainder when n is divided by 5.

Problem 5. Factor $x^4 + 1$ into two polynomials with real coefficients (Hint: x^2)

Problem 6. If $a + b + c = 0$, prove that $a^3 + b^3 + c^3 = 3abc$

Problem 7. (Challenge) The integer N is positive. There are exactly 2005 ordered pairs (x, y) of positive integers satisfying

$$\frac{1}{x} + \frac{1}{y} = \frac{1}{N}$$

Prove that N is a perfect square.