MAΘ Competition Team Problem Set 8

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Problem 1. Compute 7^{222} mod 13 using Fermat's Little Theorem. Show your work.

Problem 2. Find the least positive integer n such that

$$2^n \equiv 1 \bmod 105.$$

Problem 3. Find all n such that $\phi(n) = 20$.

Problem 4. Let $a_1, a_2, \ldots, a_{2018}$ be a strictly increasing sequence of positive integers such that

$$a_1 + a_2 + \cdots + a_{2018} = 2018^{2018}.$$

What is the remainder when $a_1^3 + a_2^3 + \cdots + a_{2018}^3$ is divided by 6?

Problem 5. Let

$$P(m) = \frac{m}{2} + \frac{m^2}{4} + \frac{m^4}{8} + \frac{m^8}{8}.$$

How many of the values $P(2022), P(2023), P(2024)$, and $P(2025)$ are integers?

Problem 6. Let N be the greatest four-digit positive integer with the property that whenever one of its digits is changed to 1, the resulting number is divisible by 7. Let Q and R be the quotient and remainder, respectively, when N is divided by 1000. Find $Q + R$.

Problem 7 (Challenge). Evaluate the power tower $\underbrace{3^{3^{3^{\cdot^{\cdot^{\cdot}}}}}}_{2012\text{ 3's}}$ modulo 100.