MAΘ Competition Team Problem Set 8

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Problem 1. Russell writes down the number 1 on the board. He repeatedly replaces the number x on the board with $2x$ or $3x$. Can Russell ever obtain the number 2025?

Problem 2. The numbers $1, 2, \ldots 100$ are written on a blackboard. You may choose any two numbers a and b and erase them, replacing them with the single number $a+b-1$. After 99 steps, only a single number will be left. What is it? What about when you replace a and b at each turn by the product $ab+a+b$?

Problem 3. A bag contains 99 red marbles and 99 blue marbles. Taking two marbles out of the bag, you:

- put a red marble in the bag if the two marbles you drew are the same color (both red or both blue)
- put a blue marble in the bag if the two marbles you drew are different colors.

Repeat this step (reducing the number of marbles in the bag by one each time) until only one marble is left in the bag. What is the color of that marble?

Problem 4. We strike the first digit of the number 7^{1996} and then add it to the remaining number. This is repeated until a number with 10 digits remains. Prove that this number has two equal digits.

Problem 5. Dom wrote down three positive real numbers on the blackboard and told Jiwu to decrease one of them by 3%, decrease another by 4%, and increase the last by 5%. Jiwu wrote down the results in his notebook. It turned out that he wrote down the same three numbers on the blackboard, just in a different order. Prove that Jiwu must have made a mistake, probably from playing too much TFT.

Problem 6. Oh no! Conflict has broken out in Davos, host city of the annual World Economic Forum that happened this week. Each attendee has at most three enemies. To prevent any further diplomatic gaffes or violent fistfights, the organizers are moving all the attendees into two buildings. Prove that the attendees can be separated such that each attendee only has at most one enemy in his own building.