MAΘ Competition Team HWSet 10

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Problem 1. Find the coordinates of the inradius and circumcenter of triangle ABC made by points $A = (0, 0)$ $B = (6, 0)$ and $C = (0, 8)$

Problem 2. Given a triangle with sides 13 and 15, and a common altitude of 12 between these side, find its inradius.

Problem 3. Also find the radius of the nine-point circle for the triangle in problem 2.

Problem 4. Isosceles triangle ABC has $AB = AC = 36$, and a circle with radius 52 is tangent to line AB at B and to line AC at C. What is the area of the circle that passes through vertices A, B, and C?

Problem 5. Prove that the orthocenter and the incenter of a triangle coincide if and only if the triangle is equilateral.

Problem 6 (Challenge). Let $\triangle ABC$ have side lengths $AB = 30$, $BC = 32$, and $AC = 34$. Point X lies in the interior of $\overline{BC}$, and points I_1 and I_2 are the incenters of $\triangle ABX$ and $\triangle ACX$, respectively. Find the minimum possible area of $\triangle AI_1I_2$ as X varies along $\overline{BC}$.

Problem 7 (Challenge). Prove that the nine-point circle exists for all triangles (for those who weren't present at the competition team lecture).