MAΘ Competition Team Problem Set 16

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April 10, 2026

For exercises 1–3, write the expression in logarithmic form.

1. $7^5 = 16807$
2. $16^{\frac{3}{4}} = 8$
3. $\left(\frac{1}{3}\right)^{-2} = 9$

For exercises 4–6, write the expression in exponential form.

4. $\log_2 32 = 5$
5. $\log_{\frac{1}{5}} \left(\frac{1}{625}\right) = 4$
6. $\log_9 \left(\frac{1}{81}\right) = -2$

Problem 1. Compute $(\log_{125} 16)(\log_4 27)(\log_3 625)$.

Problem 2. Compute

$$\frac{\log(8)}{\log\left(\frac{1}{8}\right)}.$$

Problem 3. The solution of the equation $7^{x+7} = 8^x$ can be expressed in the form $x = \log_b 7^7$. What is b?

Problem 4. If $\log(xy^3) = 1$ and $\log(x^2y) = 1$, what is $\log(xy)$?

Problem 5. Suppose that $4^a = 5$, $5^b = 6$, $6^c = 7$, and $7^d = 8$. What is $a \cdot b \cdot c \cdot d$?

Problem 6. What is the value of

$$\log_3 7 \cdot \log_5 9 \cdot \log_7 11 \cdot \log_9 13 \cdots \log_{21} 25 \cdot \log_{23} 27?$$

Problem 7 (Challenge). Determine the value of ab given

$$\log_8 a + \log_4 b^2 = 5$$

$$\log_8 b + \log_4 a^2 = 7$$